As an example we provide in Fig. 5 the dependences of ω' on φ, obtained during the numerical solution of (3.13), for the following initial conditions: $t' = 0$, $\varphi = 0.1$, $\omega' = -1; -0.5; 0$ (lines 1-3). The calculations were performed for $B = 10^{-4}$ for a plate with $D/L = 1$, corresponding to $A = 0.132 \cdot 10^{-4}$, $B_2 = -0.274 \cdot 10^{-4}$. For the same values of the original parameters we derived, with account of (3.16), the boundary (curve 4) of the region of and ω' values, in achieving which the plate is "buoyant" on the hydrofoil. The primed portions of curves 1-3, though formally corresponding to Eq. (3.13), cannot be realized. This is due to the fact that the plate behavior following the moments of "buoyancy" corresponding to the intersection points of lines 1-3 and 4 are no longer described by Eq. (3.13).

The results obtained in the examples considered for the simplest special case, when in (1.10) attention is restricted to the first equation only for each of the two $\xi(J = 1)$, are, naturally, of approximate nature. Nevertheless, it is possible to find preliminary estimates of flow characteristics. However, when higher accuracy is required, it is suggested to use in (1.10) a larger number of equations $(J \geqslant 2)$.

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PLANAR SURFACE WAVE GENERATION IN THE PRESENCE OF SLIGHT BOTTOM ROUGHNESS

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UDC 532.591

At present a linear theory of surface wave generation by various perturbations in a liquid with horizontal bottom has been developed quite well. However in the case of a liquid with rough bottom analytical studies of this problem have met with severe mathematical difficulties. The perturbation method is usually used for slight bottom roughness [1].

Using a linear formulation, the present study will investigate the effect of slight localized bottom roughness on the behavior of surface waves for two problems: decay of an initial elevation of the free surface and motion of a surface pressure region. A comparison is performed with a numerical solution of the original problem, obtained by the finite difference method.

1. Let an ideal incompressible homogeneous liquid occupy the region $-\infty < x < \infty$, $-H(x) \leqslant y \leqslant 0$, where x is the horizontal, and y, the vertical coordinate, $H(x) = H_0 - h(x)$, $h(x) \to 0$ as $|x| \to \infty$. At the initial moment $t = 0$ the free liquid surface is displaced from its equilibrium horizontal form and the expression $y = f_0(x)$ is specified. The velocity potential of the given flow $\varphi(x, y, t)$ satisfies the equation

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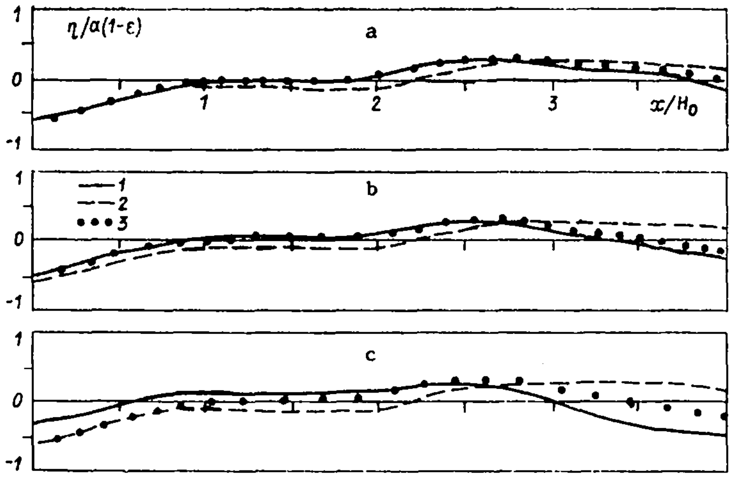

Fig. 1

$$\Delta\varphi = 0 \tag{1.1}$$

with boundary conditions

$$\varphi_{tt} + g\varphi_y = 0 \text{ for } y = 0; \tag{1.2}$$

$$\varphi_y = h_x\varphi_x \text{ for } y = -H(x); \tag{1.3}$$

$$|\varphi| < \infty \text{ for } |x| \to \infty \tag{1.4}$$

and initial conditions

$$\varphi = 0, \ \varphi_t = -gf_0(x) \text{ for } t = 0. \tag{1.5}$$

With the assumption that $h_m \ll H_0$ ($h_m = \max|h|$), in analogy to [1] the boundary condition on the bottom can be linearized, and the solution of Eqs. (1.1)-(1.5) can be sought in the form

$$\varphi = \varphi_0 + \varphi_1 + \varphi_2 + \ldots \tag{1.6}$$

The functions φ_i (i = 0, 1, 2, ...) then satisfy Eq. (1.1) and boundary conditions (1.2), (1.4). The boundary condition on the bottom, Eq. (1.3), is transferred to the line $y = -H_0$:

$$\varphi_{0y} = 0, \ \varphi_{1y} = A(x, t) \text{ for } y = -H_0$$
$$(A(x, t) = h_x\varphi_{0x} - h\varphi_{0yy}). \tag{1.7}$$

In order to find φ_i a recursive sequence of boundary problems can be derived. Further analysis will be limited to the first approximation only. We write the initial conditions of Eq. (1.5) as $\varphi_{0,1} = 0$, $\varphi_{0t} = -gf_0(x)$, $\varphi_{1t} = 0$ at t = 0. We assume that the functions $f_0(x)$ and $h(x)$ admit a Fourier transform and (for simplicity) that they are even.

The function φ_0 describes the well-known solution of this problem for a liquid with smooth bottom:

$$\varphi_0(x, y, t) = -\frac{\sqrt{2g}}{\pi}\int_0^\infty \frac{F_0(k)\cos kx \cosh k(y + H_0)}{\sqrt{k \sinh 2kH_0}} \sin\omega t \, dk$$
$$\left(\omega(k) = \sqrt{gk \tanh kH_0}, \ F_0(k) = 2\int_0^\infty f_0(x)\cos kx \, dx\right). \tag{1.8}$$

The vertical displacement of the free surface $\eta(x, t)$ is defined by the expression $\eta = -(1/g)\varphi_t|_{y=0}$, and according to Eq. (1.6),

$$\eta = \eta_0 + \eta_1. \tag{1.9}$$

Here the first term is equal to $\eta_0(x, t) = \frac{1}{\pi}\int_0^\infty F_0(k)\cos kx \cos\omega t \, dk.$

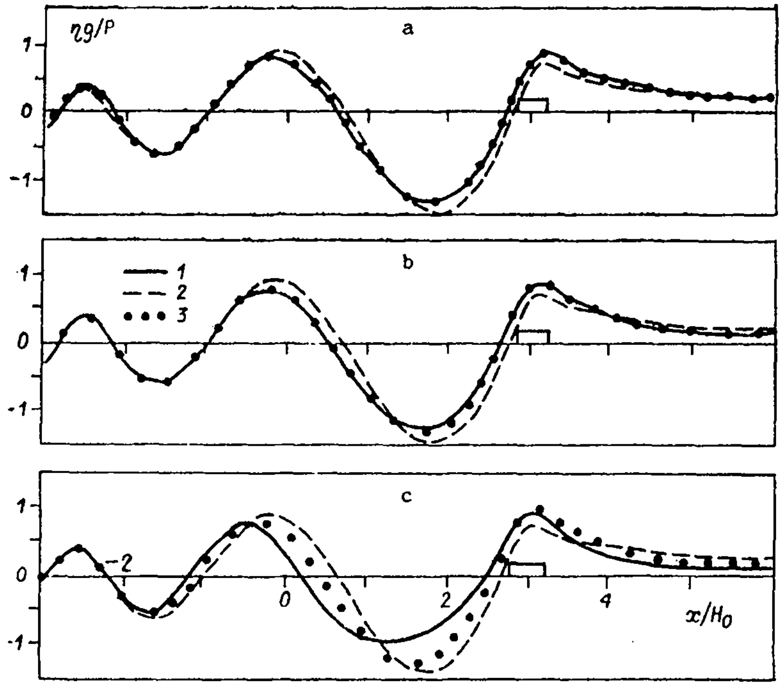

Fig. 2

To calculate the function φ_1 we use Fourier and Laplace integral transforms. According to Eqs. (1.7), (1.8)

$$A(x, t) = \frac{\sqrt{2g}}{\pi} \int_0^\infty \frac{\sqrt{k}\, F_0(k) \sin \omega t}{\sqrt{\sinh 2kH_0}} (h \sin kx)_x \, dk$$

and the solution for η_1 in Eq. (1.9) has the form

$$\eta_1(x, t) = \frac{2g}{\pi^2} \int_0^\infty \frac{\xi \cos \xi x}{\cosh \xi H_0} d\xi \int_0^\infty \frac{kF_0(k) [\cos \omega(\xi) t - \cos \omega(k) t]}{\cosh kH_0 [\omega^2(k) - \omega^2(\xi)]} K(\xi, k)\, dk$$

$$\left(K(\xi, k) = \int_0^\infty h(x) \sin kx \sin \xi x \, dx \right).$$

In a liquid layer of finite width $|x| \leqslant L$ with boundary conditions

$$\varphi_x = 0 \quad \text{at} \quad |x| = L \qquad (1.10)$$

from the solution of Eqs. (1.1)-(1.3), (1.5) in the approximation of Eq. (1.6) we find

$$\eta_0(x, t) = \frac{b_0}{2} + \sum_{n=1}^\infty b_n \cos n\beta x \cos \omega_n t; \qquad (1.11)$$

$$\eta_1(x, t) = \frac{2g\beta^2}{L} \sum_{m=1}^\infty \frac{m \cos m\beta x}{\cosh m\beta H_0} \sum_{n=1}^\infty \frac{nb_n M_{nm}}{\cosh n\beta H_0} \begin{cases} (\cos \omega_m t - \cos \omega_n t)/(\omega_n^2 - \omega_m^2) & \text{at} \quad n \neq m, \\ t \sin \omega_m t/(2\omega_m) & \text{at} \quad n = m, \end{cases} \qquad (1.12)$$

where $\beta = \pi/L$; $\omega_n = \sqrt{gn\beta \tanh n\beta H_0}$;

$$b_n = \frac{2}{L} \int_0^L f_0(x) \cos n\beta x \, dx; \quad M_{nm} = K(m\beta, n\beta).$$

2. In our study of wave motions produced by the effect of an external pressure $p_a(x,t)$ applied to the free liquid surface we will maintain the notation of Sec. 1. The velocity potential of the flow to be considered satisfies Eq. (1.1) with boundary conditions (1.3), (1.4). We now write the boundary condition on the free surface in the form

$$\varphi_{tt} + g\varphi_y = -p_{at}/\rho \quad \text{at} \quad y = 0 \qquad (2.1)$$

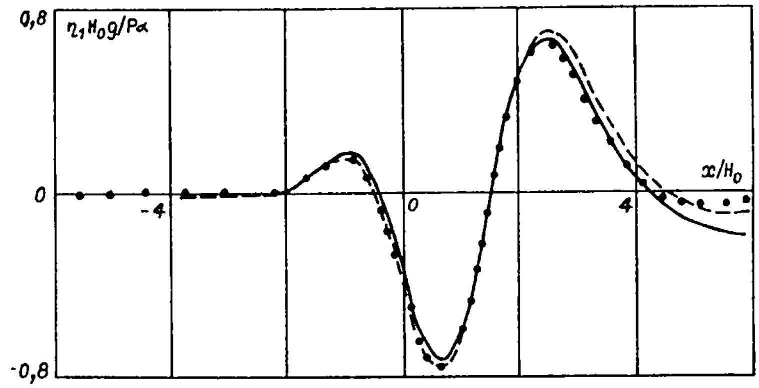

Fig. 3

(where ρ is the liquid density). We assume that at the initial moment perturbations are absent from the liquid, and the initial conditions are

$$\varphi = 0,\ \varphi_t = -p_a/\rho \quad \text{at} \quad t = 0. \tag{2.2}$$

The elevation of the free surface is determined from the expression $\eta = -(\varphi_t|_{y=0} + p_a/\rho)/g$. It is assumed that the surface pressure moves uniformly at a velocity c in the direction of the positive x-axis, which velocity is constant within the interval $|x - ct| \leqslant l$ and zero elsewhere, i.e.,

$$p_a/\rho = P \quad \text{at} \quad |x - ct| \leqslant l, \qquad p_a = 0 \quad \text{at} \quad |x - ct| > l. \tag{2.3}$$

In the approximation of slight bottom roughness the solution of the given problem may be sought in the form of Eq. (1.6), but in contrast to Sec. 1 the boundary condition on the free surface and the initial condition for the function φ_0 have the form

$$\varphi_{0tt} + g\varphi_{0y} = -p_{at}/\rho \quad \text{at} \quad y = 0,\ \varphi_{0t} = -p_a/\rho \quad \text{at} \quad t = 0.$$

All remaining conditions are the same. The solution of this problem is then:

$$\varphi_0(x, y, t) = \frac{1}{\pi}\int_0^\infty \frac{f(k)\cosh k(y + H_0)}{\cosh kH_0(\omega^2 - k^2c^2)}\{kc[\sin kx\cos\omega t - \underline{\sin k(x - ct)}] - \omega\cos kx\sin\omega t\}\,dk; \tag{2.4}$$

$$\eta_0(x, t) = \frac{1}{\pi g}\int_0^\infty \frac{f(k)\,\omega dk}{k^2c^2 - \omega^2}\{\omega[\underline{\cos k(x - ct)} - \cos\omega t\cos kx] - kc\sin\omega t\sin kx\}; \tag{2.5}$$

$$\eta_1(x, t) = \frac{1}{\pi^2}\int_0^\infty \frac{\xi d\xi}{\cosh \xi H_0}\int_0^\infty \frac{kf(k)\,dk}{\cosh kH_0}\left\{B(\omega(k)) - B(\omega(\xi)) + \frac{k^2c^2(F_1\sin kct - F_2\cos kct)}{[\omega^2(k) - k^2c^2][\omega^2(\xi) - k^2c^2]}\right\}. \tag{2.6}$$

Here

$$f(k) = 2P\sin kl/k;\ F_1(k, \xi, x) = G_1(k, \xi)\cos\xi x - G_2(k, \xi)\sin\xi x;$$

$$F_2(k, \xi, x) = G_3(k, \xi)\cos\xi x - G_1(\xi, k)\sin\xi x;$$

$$G_1(k, \xi) = \int_{-\infty}^{\infty} h(x)\sin\xi x\cos kx\,dx;\quad G_2(k, \xi) = \int_{-\infty}^{\infty} h(x)\cos\xi x\cos kx\,dx;$$

$$G_3(k, \xi) = \int_{-\infty}^{\infty} h(x)\sin\xi x\sin kx\,dx;\quad B(\omega) = \frac{\omega(kcF_1\sin\omega t - \omega F_2\cos\omega t)}{(k^2c^2 - \omega^2)[\omega^2(\xi) - \omega^2(k)]}.$$

It is interesting that in the approximation considered the effect of a varying bottom manifests itself in the following manner: in addition to wave motions produced in a liquid with a smooth bottom by the wave generator considered (initial perturbation of the free surface or a moving surface pressure region), other wave perturbations appear due to oscillation of a portion of the bottom corresponding to the size of the bottom roughness. These bottom oscillations "switch on" simultaneously with the initial generator, and their vertical velocity component can be described by a function A(x, t) which can be represented in the form $A = (hZ)_x$, where

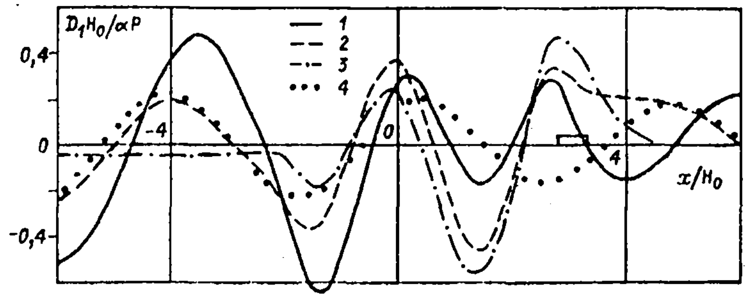

Fig. 4

$$Z(x, t) = \frac{1}{\pi}\int_0^\infty \frac{kf(k)\,dk}{\cosh kH_0\,(k^2c^2-\omega^2)}\{kc\,[\cos k(x-ct) - \cos kx\cos\omega t] - \omega\sin\omega t\sin kx\}. \tag{2.7}$$

An important characteristic of this flow is the wave resistance W experienced by a moving pressure region. With consideration of Eq. (2.3)

$$W(t) = \int_{-\infty}^{\infty} p_a(x, t)\,\eta_x\,dx = \rho P\,[\eta(ct + l, t) - \eta(ct - l, t)]. \tag{2.8}$$

For a liquid with a smooth bottom, according to Eq. (2.5),

$$W_0(t) = \frac{2\rho P^2}{\pi g}\int_0^\infty \frac{\omega}{k}\sin^2 kl\left[\frac{\sin(\omega - kc)\,t}{\omega - kc} - \frac{\sin(\omega + kc)\,t}{\omega + kc}\right]dk.$$

An asymptotic evaluation of this expression at $t \to \infty$ yields a solution of the corresponding steady state problem, as presented, for example, in [2]. $W_0 \approx 4\rho P^2\sin^2 k_0 l/g(1 - gH_0/c^2\cosh^2 k_0H_0)$ (where k_0 is the root of the equation $c^2k = g\tanh kH_0$). The presence of slight roughness introduces into W in addition to W_0 a term W_1 dependent on the behavior of the function η_1 in Eq. (2.6):

$$W_1 = \frac{2gP}{\pi^2}\int_0^\infty \frac{\xi\sin\xi l\,d\xi}{\cosh\xi H_0}\int_0^\infty \frac{kf(k)\,dk}{\cosh kH_0}\left\{C(\omega(k)) - C(\omega(\xi)) + \frac{k^2c^2(E_1\cos kct - E_2\sin kct)}{[\omega^2(k) - k^2c^2]\,[\omega^2(\xi) - k^2c^2]}\right\}, \tag{2.9}$$

where $C(\omega) = \dfrac{\omega(\omega E_1\cos\omega t - kcE_2\sin\omega t)}{(k^2c^2 - \omega^2)\,[\omega^2(\xi) - \omega^2(k)]}$;

$$E_1(k, \xi, t) = G_3(k, \xi)\sin\xi ct + G_1(k, \xi)\cos\xi ct;$$
$$E_2(k, \xi, t) = G_1(k, \xi)\sin\xi ct + G_2(k, \xi)\cos\xi ct.$$

For a liquid layer of finite width $|x| \leqslant L$ the solution for motion of the surface pressure region $p_a/\rho = P$ at $|x + x_0 - ct| \leqslant l$, $p_a = 0$ at $|x + x_0 - ct| > l$ with consideration of boundary condition (1.12) has the form

$$\eta_0 = \frac{2P}{\pi g}\sum_{n=1}^\infty T_n\cos n\beta(x + L); \tag{2.10}$$

$$\eta_1 = \sum_{m=1}^\infty N_m\cos m\beta(x + L). \tag{2.11}$$

Here

$$T_n = \frac{\omega_n\sin n\beta l}{n(\omega_n^2 - n^2\beta^2c^2)}[\omega_n\cos n\beta(x_0 - L)(\cos\omega_n t - \cos n\beta ct) +$$
$$+ \sin n\beta(x_0 - L)(n\beta c\sin\omega_n t - \omega_n\sin n\beta ct)];$$

$$N_m = \frac{2\pi mP}{L^3\cosh m\beta H_0}\sum_{n=1}^\infty \frac{B_{nm}}{\cosh n\beta H_0}\left[G(\omega_n) - G(\omega_m) + \frac{n^2\beta^2c^2\cos n\beta(x_0 - L - ct)}{(n^2\beta^2c^2 - \omega_n^2)(\omega_m^2 - n^2\beta^2c^2)}\right];$$

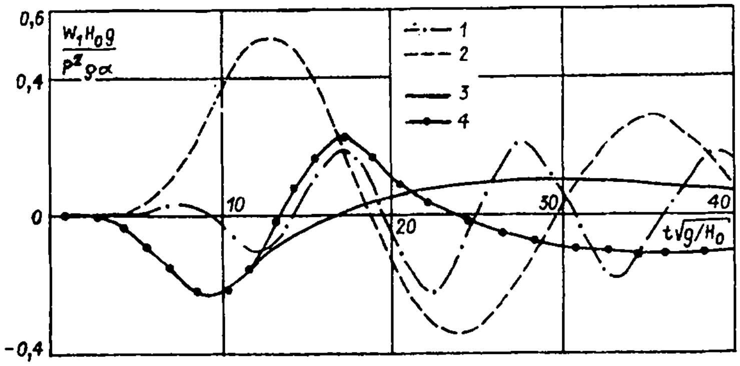

Fig. 5

$$B_{nm} = \int_{-L}^{L} h(x) \sin n\beta (x + L) \sin m\beta (x + L)\, dx;$$

$$G(\omega) = \frac{\omega\,[n\beta c \sin n\beta (x_0 - L) \sin \omega t + \omega \cos n\beta (x_0 - L) \cos \omega t]}{(\omega^2 - n^2\beta^2 c^2)(\omega_m^2 - \omega_n^2)}.$$

The wave resistance in the given problem (according to Eqs. (2.8), (2.10), (2.11)) $W = W_0 + W_1$, where

$$W_0 = \frac{4\rho P^2}{\pi g} \sum_{n=1}^{\infty} T_n \sin n\beta l \cdot \sin n\beta (x_0 - ct - L),$$

$$W_1 = 2gP \sum_{m=1}^{\infty} N_m \sin m\beta l \cdot \sin m\beta (x_0 - ct - L).$$

In addition to these flow characteristics we will also consider the behavior of pressure on the bottom, which can be determined by the expression $D = \rho^{-1} p|_{y=-H_0} = -\varphi_t|_{y=-H_0}$. For a liquid with a smooth bottom

$$D_0 = \frac{lP}{L} + \frac{2P}{\pi} \sum_{n=1}^{\infty} R_n \cos n\beta (x + L),$$

$$R_n = \frac{\sin n\beta l}{n \cosh n\beta H_0 \left(\omega_n^2 - n^2\beta^2 c^2\right)} [\omega_n^2 \cos \omega_n t \cos n\beta (x_0 - L) - n^2\beta^2 c^2 \cos n\beta (x_0 - L - ct) + n\beta c \omega_n \sin \omega_n t \sin n\beta (x_0 - L)].$$

In the approximation of slight roughness, to the term D_0 we add

$$D_1 = \sum_{m=1}^{\infty} V_m \cos m\beta (x + L), \qquad (2.12)$$

$$V_m = \frac{2P}{L^2} \sum_{n=1}^{\infty} \frac{\sin n\beta l B_{nm}}{\cosh n\beta H_0} \left\{ (gm\beta - \omega_n^2 \tanh m\beta H_0) [G(\omega_m) - G(\omega_n)] + \frac{n^2\beta^2 c^2 \left(n^2\beta^2 c^2 \tanh m\beta H_0 - gm\beta\right)}{\left(\omega_n^2 - n^2\beta^2 c^2\right)\left(\omega_m^2 - n^2\beta^2 c^2\right)} \cos n\beta (x_0 - L - ct) \right\}.$$

The effect of bottom roughness on a steady state wave train is also of significant interest. In this case the solutions for $\varphi_0(x, t)$ and $\eta_0(x, t)$ are equal to the underlined terms in Eqs. (2.4), (2.5) and in a moving coordinate system $x_* = x - ct$ they describe wave perturbations produced by a constantly acting pressure applied to the free surface of a uniform flow. The solution of this problem is well known (see, e.g., [3]). The value of the function $Z(x, t)$ is defined by the underlined terms in Eq. (2.7). According to [3], at $c > \sqrt{gH_0}$ the function $Z(x_*)$ is even with respect to x_* and decays exponentially with removal from the external pressure region. For $c < \sqrt{gH_0}$ together with this term there appears a second one which is nonzero only for $x_* < 0$ and represents a periodic wave. The rate of decay of the function $Z(x_*)$ with removal from the pressure region for $x_* > 0$ permits determination of the minimum distance from the bottom roughness at which the latter ceases to have a significant effect. Beginning at those times when the surface pressure falls within the roughness zone the bottom excitation function A(x, t) becomes periodic in time, and the problem under study has much in common with the problem of wave generation by periodic bottom movements. In this case the solution for $\varphi_1(x, t)$ has the form

$$\varphi_1 = \frac{c}{\pi^2}\int_0^\infty \frac{k^2 f(k)\,dk}{\cosh kH_0\,[k^2c^2-\omega^2(k)]}\int_0^\infty \frac{d\xi}{\cosh \xi H_0\,[\omega^2(\xi)-k^2c^2]}\times$$

$$\times\left\{[\omega^2(\xi)\sinh \xi y + g\xi\cosh \xi y]\left[F_1\cos\omega(\xi)t + kcF_2\frac{\sin\omega(\xi)t}{\omega(\xi)}\right] - (k^2c^2\sinh\xi y + g\xi\cosh\xi y)(F_1\cos kct + F_2\sin kct)\right\}.$$

3. An algorithm for numerical solution of the problems considered above can be constructed on the basis of the finite difference method for a finite width liquid layer. We use the replacement of variables $x' = x$, $y' = -y/H(x)$, which transforms the flow region $Q = \{|x| \leqslant L, -H(x) \leqslant y \leqslant 0\}$ into a rectangle $\Pi = \{|x'| \leqslant L, 0 \leqslant y' \leqslant 1\}$. As a result of this replacement Eq. (1.1) and boundary conditions (1.2), (1.3), (1.12), (2.1) take on the form (here and below we omit the primes on the new variables)

$$u_x + v_y = 0; \tag{3.1}$$

$$u = 0 \text{ at } |x| = L,\ v = 0 \text{ at } y = 1; \tag{3.2}$$

$$\eta_t = -v,\ \varphi_t = -g\eta - p_a/\rho \text{ at } y = 0; \tag{3.3}$$

$$u = (\varphi_x + \beta v)/\gamma,\ v = (\varphi_y + \beta u)/H(x),\quad \beta = -yH_x,\ \gamma = (1+\beta^2)/H(x). \tag{3.4}$$

Initial conditions (1.5), (2.2) remain as before. After writing evolutionary equations (3.3) in discrete form using the Crank—Nicholson method [4] the problems of finding φ and η are easily separated (within the limits of a single step in time). From Eqs. (3.1), (3.2), (3.4) and the boundary condition

$$\varphi - g\tau^2 v/4 = a \text{ at } y = 0 \tag{3.5}$$

we calculate φ, u, v, after which direct computation using $\eta = -\tau v/2 + b$ at $y = 0$ yields the position of the free surface. Here τ is the step in time, and a and b denote all terms known from the previous time step (or from initial data) and the specified function $p_a(x, t)$:

$$a^{n+1} = \varphi^n + g\tau^2 v^n/4 - \tau\left[g\eta^n + (p_a^{n+1} + p_a^n)/2\rho\right],\quad b^{n+1} = \eta^n - \tau v^n/2$$

(with superscript denoting the number of the step in t). The problem of Eqs. (3.1), (3.2), (3.4), (3.5) can be calculated by an iterative splitting technique constructed using the principle of the stabilizing correction method [5] with consideration of the divergent form of Eq. (3.1):

$$(\varphi^{k+1/2} - \varphi^k)/\omega = u_x^k + v_y^{k+1/2},\quad v^{k+1/2} = (\varphi_y^{k+1/2} + \beta u^k)/H(x),$$

$$\varphi^{k+1/2} - g\tau^2 v^{k+1/2}/4 = a \text{ at } y = 0,\quad v^{k+1/2} = 0 \text{ at } y = 1;$$

$$(\varphi^{k+1} - \varphi^{k+1/2})/\omega = u_x^{k+1} - u_x^k,\quad u^{k+1} = (\varphi_x^{k+1} + \beta v^{k+1/2})/\gamma,$$

$$u^{k+1} = 0 \text{ at } |x| = L$$

(where ω is the iteration parameter and the superscript is the number of the iteration). The technique can be realized on a "chessboard" grid with nodes φ located at the centers of the grid cells, nodes u at the midpoints of the side faces, and nodes v at the midpoints of the upper and lower faces. Only symmetric differences are used to approximate the derivatives. The technique developed has second order approximation for all variables and is absolutely stable and conservative.

4. For concrete calculations of the problem considered in Sec. 1 we will specify the bottom roughness shape in the form $h(x) = \alpha\cos(\pi x/2x_1)$ for $|x| \leqslant x_1$, $h(x) = 0$ for $|x| > x_1$. Cases of both finite and infinite liquid layer width were considered. A comparison with numerical calculations of the given problem by the finite difference method described in Sec. 3 was carried out for a liquid layer of finite width $L/H_0 = 4$ for an initial elevation of the free surface $f_0(x) = a[\cos(\pi x/2r_0) - \varepsilon]$ at $|x| \leq x_0$, $f_0(x) = -a\varepsilon$ for $|x| > x_0$ ($\varepsilon = 2x_0/\pi L$). The numerical solution was obtained on a 72 × 12 grid with time step $\tau\sqrt{g/H_0} = 0.02$.

Figure 1 shows the quantity η for times $t\sqrt{g/H_0} = 30$ at $x_0/H_0 = 1$, $x_1/H_0 = 2$, $\alpha/H_0 = 0.2$ (a); $x_1/H_0 = 3$, $\alpha/H_0 = 0.2$ (b); $x_1/H_0 = 3$, $\alpha/H_0 = 0.3$ (c). Curves 1-3 represent the numerical solution, the solution of Eq. (1.11) for a smooth bottom, and the approximate solution of Eq. (1.9) for slight roughness, obtained with use of Eqs. (1.11), (1.12). It is obvious that the approximate solution coincides with the numerical one only for $\alpha/H_0 \le 0.2$. It should be noted that in the presence of vertical walls reflections cause the effect of bottom roughness to be significantly more intense than in an unbounded liquid.

To perform calculations of the problem considered in Sec. 2 two types of bottom roughness were used:

$$h(x) = \alpha \cos(\pi(x - x_1)/2x_2) \text{ for } |x - x_1| \leqslant x_2, \quad h(x) = 0 \text{ for } |x - x_1| > x_2; \tag{4.1}$$

$$h(x) = \alpha \sin(\pi(x_1 - x)/x_2) \text{ for } |x - x_1| \leqslant x_2, \quad h(x) = 0 \text{ for } |x - x_1| > x_2. \tag{4.2}$$

Comparison with a numerical solution was carried out for the roughness of Eq. (4.1) for $L/H_0 = 6$, $l/H_0 = 0.25$, $x_0/H_0 = 5$, $c/\sqrt{gH_0} = 0.8$, $x_1 = 0$, $x_2/H_0 = 3$. Figure 2 shows vertical displacements of the free surface at times $t\sqrt{g/H_0} = 10$ for $\alpha/H_0 = 0.2$ (a), $\alpha/H_0 = 0.3$ (b), $\alpha/H_0 = 0.5$ (c). The numerical calculations were performed on a 100 × 10 grid with a time step $\tau\sqrt{g/H_0} = 0.02$. Curves 1-3 are analogous to Fig. 1, with the rectangle being the position of the pressure region at the given time. The behavior of the function η_1 found from Eq. (2.11) at $t\sqrt{g/H_0} = 10$, is shown in Fig. 3, where curve 1 corresponds to the roughness of Eq. (4.1) with parameters of Fig. 2, curve 2 is the same, but for $L/H_0 = 20$, and curve 3 is the roughness of Eq. (4.2) ($L/H_0 = 20$, $x_1/H_0 = 3$, $x_2/H_0 = 6$, other parameters the same). The behavior of the function η_1 for $L/H_0 = 20$ at the given time practically coincides with the case of the unbounded liquid. It is evident that change in the width of the liquid layer and roughness shape affect the behavior of the free surface only insignificantly. However the behavior of pressure on the bottom differs greatly in these cases.

Figure 4 shows values of D_1 calculated with Eq. (2.12). Curves 1-3 are analogous to Fig. 3, while curve 4 corresponds to the roughness of Eq. (4.1) located to the left of the initial position of the pressure region ($L/H_0 = 20$, $x_1/H_0 = -9$, $x_2/H_0 = 3$, other parameters as before). It is evident that the bottom roughness causes significant perturbations of the pressure, while under the given conditions changes in vertical displacements are negligibly small (the maximum values of $|\eta_1| gH_0/\alpha P < 0.01$). This is true because instantaneous propulsion of the pressure region causes perturbations described by the functions $Z(x, t)$ in Eq. (2.7) propagating in both positive and negative x-directions.

The effect of bottom roughness on wave resistance is shown in Fig. 5, which shows the functions W_1 for an infinite liquid, as defined by Eq. (2.9). Curves 1-3 correspond to roughness of the form of Eq. (4.1) with parameters used in Fig. 3, and various rates of motion of the pressure region: $c/\sqrt{gH_0} = 0.4$; 0.6; 0.8. Line 4 is for the roughness of Eq. (4.2) with the same parameters used for curve 3 of Fig. 3 ($c/\sqrt{gH_0} = 0.8$). It is evident that the presence of bottom roughness changes wave resistance not only for times corresponding to passage of the pressure region above the roughness, but also at significantly later times. The function W_1 is then a damped wave, the period of which is determined solely by the rate of motion of the pressure region.

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